

Exponential functions (Half-life)		
1	<p>The amount <math>A_t</math> micrograms of a certain radioactive substance remaining after <math>t</math> years decreases according to the formula <math>A_t = A_0e^{-0.003t}</math>, where <math>A_0</math> is the amount present initially</p> <p>Find the half-life of this substance</p>	4
2	<p>Polonium P-210 is a radioactive substance which decays according to the law <math>M_t = M_0e^{-0.005t}</math>, where <math>M_0</math> is the initial mass and <math>M_t</math> is the mass remaining after <math>t</math> days</p> <p>Determine the half-life of Polonium P-210</p>	4
3	<p>The concentration of a pesticide in soil can be modelled by the equation <math>P_t = P_0e^{-0.028t}</math>.</p> <p>Where <math>P_0</math> is the initial concentration,  <math>P_t</math> is the concentration at time <math>t</math>  <math>t</math> is time in days after the application of the pesticide</p> <p>Determine the half-life of this pesticide</p>	4
4	<p>A small meteor passes through a dust shower. It picks up particles and gains weight (kg) in time <math>t</math> (hours) according to the expression <math>W_t = 1.2e^{0.06t}</math></p> <p>(a) What was the initial weight of the meteor</p> <p>(b) How long will it take for the meteor to double in weight</p>	1 4
5	<p>The number of bacteria in a sample being monitored is increasing according to the formula <math>B_t = 100e^{0.7t}</math>. <math>B_t</math> is the number of bacteria after <math>t</math> hours.</p> <p>(a) Calculate the number of bacteria present at the start of the monitoring process</p> <p>(b) How long, in hours and minutes, will it take for the sample to triple in size</p>	1 5

Exponential Half-Life - Answers		
1	<p>Set up a half-life equation i.e.</p> <p>Take natural logs of both sides</p> <p>Simplify</p> <p>Solve to find <math>t</math></p>	$1 = 2e^{-0.003t}$ $\frac{1}{2} = e^{-0.003t}$ $\log_e\left(\frac{1}{2}\right) = \log_e e^{-0.003t}$ $\log_e\left(\frac{1}{2}\right) = -0.003t$ $\frac{\log_e(1/2)}{-0.003} = t, t = 231 \text{ years}$
2	<p>Set up a half-life equation i.e.</p> <p>Take natural logs of both sides</p> <p>Simplify</p> <p>Solve to find <math>t</math></p>	$1 = 2e^{-0.005t}$ $\frac{1}{2} = e^{-0.005t}$ $\log_e\left(\frac{1}{2}\right) = \log_e e^{-0.005t}$ $\log_e\left(\frac{1}{2}\right) = -0.005t$ $\frac{\log_e(1/2)}{-0.005} = t, t = 138.6 \text{ days}$
3	<p>Set up a half-life equation i.e.</p> <p>Take natural logs of both sides</p> <p>Simplify</p> <p>Solve to find <math>t</math></p>	$1 = 2e^{-0.028t}$ $\frac{1}{2} = e^{-0.028t}$ $\log_e\left(\frac{1}{2}\right) = \log_e e^{-0.028t}$ $\log_e\left(\frac{1}{2}\right) = -0.028t$ $\frac{\log_e(1/2)}{-0.028} = t, t = 24.8 \text{ days}$
4	<p>Substitute <math>t = 0</math> into the equation</p> <p>Set up an equation i.e.</p> <p>Take natural logs of both sides</p> <p>Simplify</p> <p>Solve to find <math>t</math></p>	$W_t = 1.2e^0, W_t = 1.2 \text{ Kg}$ $2.4 = 1.2e^{0.06t}$ $2 = e^{0.06t}$ $\log_e(2) = \log_e e^{0.06t}$ $\log_e(2) = 0.06t$ $\frac{\log_e(2)}{0.06} = t, t = 11.55 \text{ hours}$
5	<p>Substitute <math>t = 0</math> into the equation</p> <p>(b) Set up an equation i.e.</p> <p>Take natural logs of both sides</p> <p>Simplify</p> <p>Solve to find <math>t</math></p> <p>Answer in hours and minutes</p>	$B_t = 100e^0, B_t = 100$ $300 = 100e^{0.7t}$ $3 = e^{0.7t}$ $\log_e(3) = \log_e e^{0.7t}$ $\log_e(3) = 0.7t$ $\frac{\log_e(3)}{0.7} = t, t = 1.5694 \text{ hours}$ $1 \text{ hour and } 34 \text{ minutes}$